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Pairing in exotic and in stable nuclei

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Abstract. The exchange of low-lying collective vibrations between pairs of nucleons moving in time reversal states close to the Fermi energy provides a conspicuous contribution to the nuclear pairing interaction, which accounts for 30–50% of the pairing gap in the case of nuclei along the stability valley, and to essentially all of the pairing correlations of the most loosely bound nucleons in the case of halo nuclei.

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1 Introduction

Discussing the free (bare) electron mass and the associated self-energy (cf. fig. 1(f)), Feynman states [1]: "Now nothing is really free ... making measurements at t_1 and t_3 we could not tell if the electron has emitted or absorbed any number of photons ... . Physically, this means that what we measure (the "experimental" mass, $m_{\rm exp}$) is not the "bare" mass but something else which includes the effect of virtual processes ... . This discussion shows that the bare mass is in fact not direcly observable".

The whole program of renormalizing the electron as well as the photon and the electron-photon coupling is known as quantum electrodynamics (cf., *e.g.*, [2] as well as fig. 1). The theory of quantum electrodynamics is also the paradigm of theories which attempt at explaining nuclear phenomena, in particular the nuclear field theory (NFT) [3–8].

In this contribution we shall show that implementing the NFT program to the highest possible degree of accuracy, trying to parallel QED, a consistent and accurate description of the nuclear structure is achieved. To do this, we use the formalism of the Dyson equation [9] which provides a complete description of the single-particle motion, and thus of the nuclear structure at large (cf. fig. 1(k)).

Theory produces state-dependent observables (effective masses, charges, spectroscopic factors, gaps, etc). Average values of these quantities for states around the Fermi energy will be used to carry out comparison between theory and experiment.

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2 Results

2.1 Stable nuclei

The program described in fig. 1 has been carried out for the case of the ^{120}Sn and for the odd isotopes ^{119}Sn and ^{121}Sn. Results are displayed in figs. 2, 3 and 4.

Concerning the pairing gap, the Hartree-Fock-Bogoliubov theory, making use of a mean field (figs. 1(e1) and (e2)) provided by a Skyrme-like interaction ($m_k = 0.7m$) in the particle-hole channel, and by the Argonne nucleon-nucleon potential in the particle-particle channel (fig. 1(b)) leads to a value of $\Delta = \langle \Delta_\nu \rangle$ (averaged over the single-particle states close to the Fermi energy) of 0.75 MeV, as compared to an experimental value of 1.4 MeV (fig. 2). Taking into account the coupling to surface vibrations (fig. 1(c)-(d)) theory essentially coincides with experiment (cf. also [10,11]).

It is to be noted that the strong deviation found in the value of $\Delta_{d_{5/2}}$ with respect to the other values is connected with the strong fragmentation this quasiparticle state undergoes due to the coupling to low-lying collective vibrational states. The renormalization provided by NFT is correct. What is not correct is its translation in terms of a single pole (quasiparticle approximation).

The renormalized quasiparticle states (fig. 1(g)) are close in energy to the experimental values (cf. fig. 3) as well as the energy and $B(E2)$ value (fig. 1(h)-(j)) associated with the renormalized lowest 2^+ state of ^{120}Sn (cf. fig. 4).

One could argue that similar results for each of the individual quantities can be obtained making use of effective interactions (Gogny, Skyrme, etc.) and standard

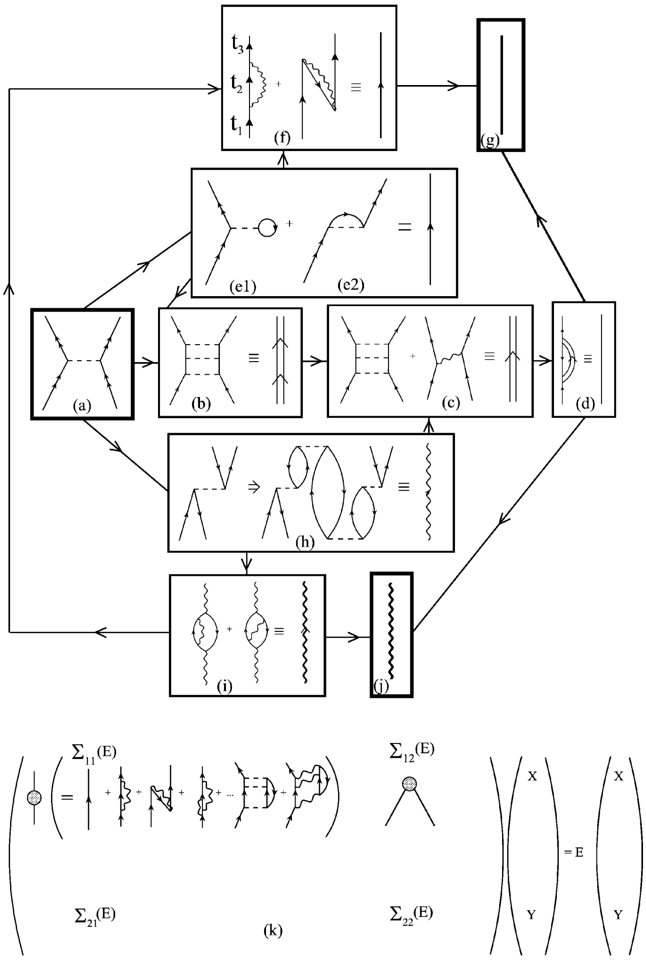

Fig. 1. The basic renormalization scheme of NFT. (a) Starting from the bare nucleon-nucleon potential v_{12} and masses represented by a horizontal dashed line and by double-arrowed lines, respectively, a mean field containing a direct (e1) (Hartree) and an exchange (Fock) term (e2) is found, the corresponding solution (single-particle) being represented in terms of a single-arrowed line to which the so-called k-mass ($m_k \approx 0.7m$, m being the bare nucleon mass) is associated. Within the corresponding particle-hole basis, v_{12} gives rise to correlated particle-hole excitations (vibrations) which, in the harmonic approximation (random phase approximation (RPA)), correspond to phonons indicated by an arrowed wavy line (h). It also gives rise to particle-particle correlations which diagonalized in the RPA lead to correlations and to pairing vibrations (and essentially a pairing gap) represented by a double-arrowed line (b). The coupling of correlated particle-hole vibrations leads to dressed single-particle states (single-arrowed heavy line (f)) and to an ω-mass ($m_\omega \approx (1+\lambda)m$, $\lambda \approx 0.5$) resulting in an effective mass $m^* = m_k m_\omega / m \approx m$ and in a spectroscopic factor $Z_\omega = 1/m_\omega$. Taking into account the exchange of vibrations between particles moving in time reversal states close to the Fermi energy leads to a renormalization of the pairing interaction and of the pairing gap and pairing vibrations (single-arrowed double line) (c) and to quasiparticles; quasiparticles which, when dressed by vibrations, describe the fermion properties (neutrons and protons) (g), which can be compared with the experimental data, in a way similar to that in which the renormalized vibrations (i) (single-arrowed wavy line) lead, in the quasiparticle basis, to the physical vibrations (j) (heavy-drawn wavy line), whose properties can be compared directly with the data. Consequently, the points of contact between theory and experiment are the bare nucleon-nucleon potential (a) through the nucleon-nucleon phase shifts, the quasiparticles properties (g) (energies, lifetimes, spectroscopic factors, effective charge masses, etc.) and the properties of the dressed vibrations (j). To carry out this program, use can be made of the NFT iterating all the relevant processes arising from four-point vertices (v_{14}) and from the particle-vibration coupling mechanism making use of the Dyson equations in the version of ref. [9], containing the contribution to the particle self-energy arising from normal Σ_{11} and abnormal (pairing) self-energies Σ_{12} (cf. (k)).

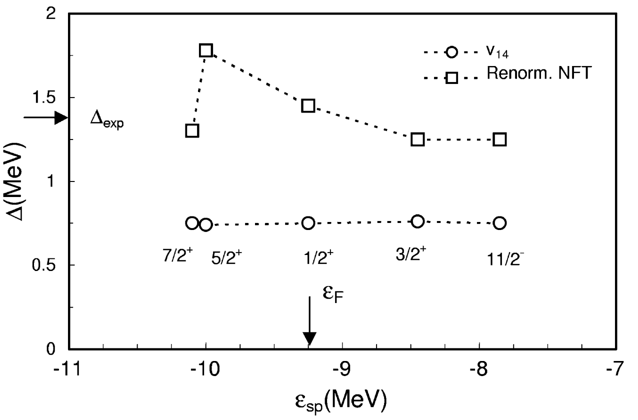

Fig. 2. State-dependent pairing gap for ^{120}Sn. The open circles indicate the BCS solution calculated making use of a mean field calculated with a Skyrme-type interaction ($m_k \approx 0.7m$, cf. fig. 1(e1) and (e2)) and a residual v_{14} Argonne nucleon-nucleon potential ($^1S_0$ channel). The open squares indicate the values of the solutions of Dyson equation (fig. 1(k)), which take into account the renormalization effects arising from the particle-vibration coupling. Also indicated are the empirical value of the pairing gap Δ_{exp} and the Fermi energy ϵ_{F}.

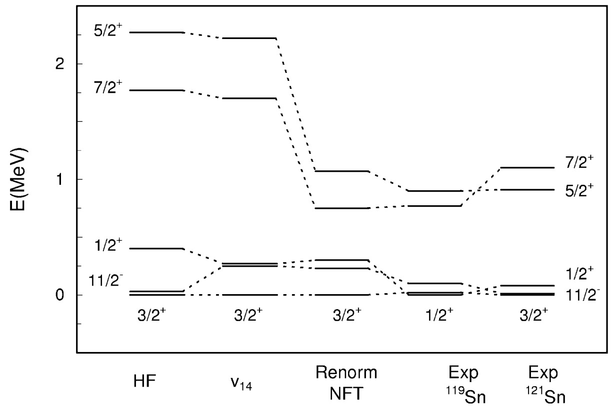

Fig. 3. Quasiparticle energies close to the Fermi energy calculated in the Hartree-Fock (HF) approximation making use of a Skyrme interaction ($m_k = 0.7m$) and diagonalizing the v_{14} Argonne nucleon-nucleon potential in the BCS approximation. The results of the self-consistent solution which diagonalizes both v_{14} plus renormalization and induced interaction effects (cf. fig. 1(k)) are also shown (Renorm. NFT) in comparison with the experimental findings.

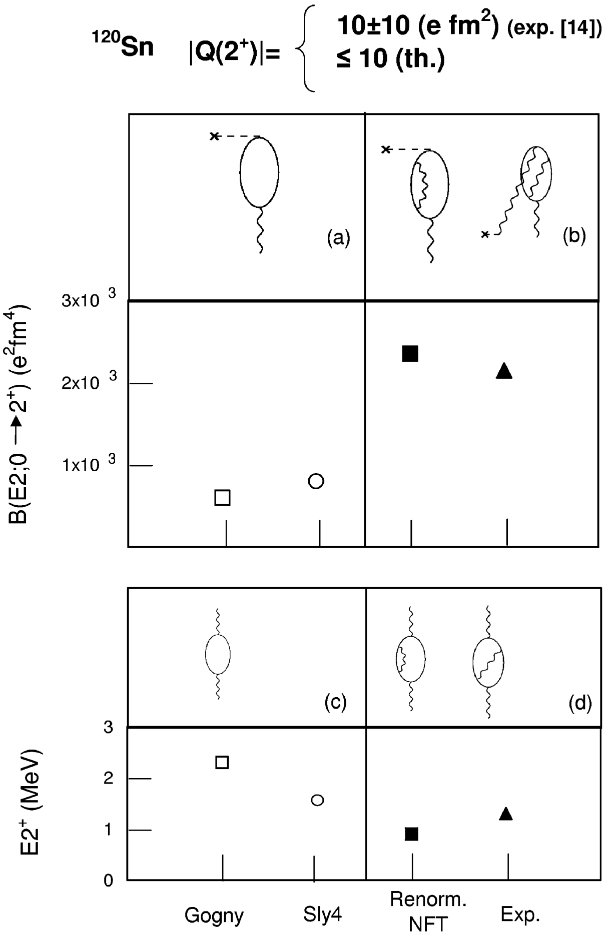

Fig. 4. The energy and the transition probability associated with the lowest 2^+ state of ^{120}Sn calculated making use of Gogny (empty squares) and Skyrme (empty circles) forces in QRPA (processes (a) and (c)). Renormalized values taking into account the coupling to intermediate states containing two quasiparticle states and a collective vibration (processes (b) and (d)) are shown by filled squares in comparison with experimental findings depicted by filled triangles. Also given is the calculated static quadrupole moment in NFT (renormalized through particle-vibration coupling) in comparison with the data.

mean-field theory. This is correct. What cannot be obtained with a single parametrization of an effective interaction are all the different quantities at once nor their frequency and temperature dependence (cf., *e.g.*, [12,13]). Furthermore, the renormalization processes arising from the interweaving of particles and vibrations are not an optional but an essential part of a consistent theory of the nuclear structure and thus are to be taken into account in all cases (cf. also [1]).

2.2 Exotic nuclei

Some of the basic ingredients which characterize light exotic nuclei are parity inversion and breaking of shells. Such phenomena are accurately accounted for in terms of NFT making use of the v_{14} nucleon-nucleon Argonne potential and of the induced pairing interaction as can be seen from tables 1 and 2 [15,16]. Of particular relevance is the fact that, in the present case, pairing correlations are essentially all due to the exchange of collective vibrations. This is connected with two facts: a) the small ℓ-space (s, p, d) available to the two loosely bound nucleons (^{11}Li, ^{12}Be) to correlate, which does not allow them to profit from the strong high-ℓ $^1S_0$ scattering process with positive phase shifts associated with v_{14}; b) the long wavelength softness of the linear response of these very extended, highly polarizable systems (characterized by a large nucleon spill out).

Table 1. Single-particle energies associated with the states s and p in ^{10}Li. For ^{11}Li the table lists the two-neutron separation energy S_{2n}, the amplitude of the s^2 and of p^2 configurations in the ground-state wave function, the mean square radius $\langle r^2 \rangle^{1/2}$ and the full width $\Delta p_\perp$ of the momentum distribution of the neutrons emitted in the direction perpendicular to the beam during the breakup of ^{11}Li [15].

		Experiment	Theory	
			Particle-vibration +Argonne	Mean field
$^{10}_{3}\text{Li}_7$ (not bound)	s	0.1–0.2 MeV	0.2 MeV (virtual)	$\approx$ 1 MeV (virtual)
	p	0.5–0.6 MeV	0.5 MeV (res.)	-1.2 MeV (bound)
$^{11}_{3}\text{Li}_8$ (bound)	S_{2n}	0.294 ± 0.03 MeV	0.33 MeV	2.4 MeV
	s^2, p^2	50% , 50%	40% , 58%	0% , 100%
	$\langle r^2 \rangle^{1/2}$	3.55 ± 0.1 fm	3.75 fm	2.1 fm
	$\Delta p_\perp$	48 ± 10 MeV/c	55 MeV/c	$\approx$ 150 MeV/c

Table 2. Single-particle energies associated with the states s, p and d in ^{11}Be and the associated spectroscopic factors for the removal of one neutron. For ^{12}Be the table lists the two-neutron separation energy S_{2n}, the amplitude of the s^2, p^2 and of d^2 configurations and the spectroscopic factors the for the removal of one neutron (calculations from [15] and [16] while experimental data from [17] and [18]).

		Experiment	Theory	
			Particle-vibration +Argonne	Mean field
$^{11}_{4}\text{Be}_7$	$E_{s_{1/2}}$	-0.504 MeV	-0.48 MeV	$\sim$ 0.14 MeV
	$E_{p_{1/2}}$	-0.18 MeV	-0.27 MeV	-3.12 MeV
	$E_{d_{5/2}}$	resonant state	$\sim$ 0 MeV	$\sim$ 2.4 MeV
	$S\left[1/2^+\right]$	0.77	0.87	0
	$S\left[1/2^-\right]$	0.96	0.86	1
$^{12}_{4}\text{Be}_8$	S_{2n}	3.673 MeV	3.6 MeV	6.24 MeV
	s^2, p^2, d^2		23%, 29%, 48%	0%, 100%, 0%
	$S\left[1/2^+\right]$	0.42 ± 0.10	0.31	0
	$S\left[1/2^-\right]$	0.36 ± 0.10	0.57	1

2.3 Conclusion

The renormalization program of nuclear field theory leads, in all cases in which it has been systematically applied, to an accurate, microscopic description of the experimental findings, providing a self-consistent and economic unified description of single-particle and of collective degrees of freedom. It is found, among other things, that the exchange of low-lying collective vibrations accounts for 50% of the pairing gap in stable nuclei and for essentially all of the pairing correlations in light exotic nuclei.

In view of these results, a serious rethinking concerning the way pairing in nuclei, in particular in exotic nuclei, is treated (calculated) is called for.

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